Efficient and Secure Compression Functions for Arithmetization-Oriented Hashing

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Verifiable Computation, Blockchains, and ZK-SNARKs

Verifiable Computation and ZK-SNARKs

Verifiable Computation for Trusted Cloud/P2P:

- Server: computes some function F, makes use of secret data.
- Clients: verify the correctness of the results.
- Use ZK-SNARKs:
  - Server $\iff$ Prover, Clients $\iff$ Verifiers
- Virtual Machines, Blockchains, Recursive SNARKs...

Hash functions and ZK-SNARKs

Hash functions play a central role:

- Blockchain roll-ups involve Merkle Tree (MT) hashing...
- ...And so does verification of recursive proofs.
- MT as commitment scheme $\Rightarrow$ **opening proof**.

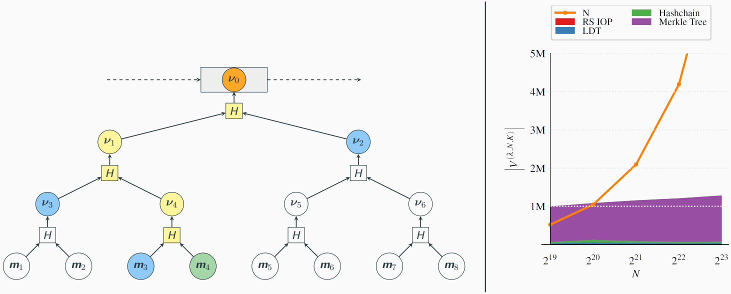


Figure 1: Left: binary Merkle Tree. Right: Fractal [6] verifier.

Arithmetization-Oriented Hash Functions

Computation complexity of a ZK-SNARK protocol:

- Proof Verification is fast (often constant time).
- Generation depends on the hash **multiplicative complexity**:
  - ◇ arithmetic circuit over a large (64/256-bits) prime field $\mathbb{F}_p$.
- Bit-oriented hash functions have high mult. complexity.
  - ◇ Bitwise operations in terms of field addition/multiplication.
- Arithmetization-oriented hash functions: defined over $\mathbb{F}_p$.
  - ◇ We will consider Poseidon [8] as an example.

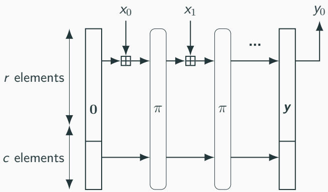
Primitive	Underlying Field	Native evaluation time	Proof generation time
SHA-256	$\mathbb{F}_2$	≈ 1	≈ 1000
Poseidon	$\mathbb{F}_p$	≈ 10	≈ 1

The PGV-LC and PGV-ELC Modes of Compression

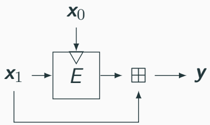
Blockcipher/Permutation-based hash functions

Compositional paradigms to obtain provable security guarantees:

- Permutation-based, like Sponge, used in SHA-3, POSEIDON.
    - ◇ Permutation is often a fixed-key blockcipher.
    - ◇ Provably secure over $\mathbb{F}_p$ (SAFE [12]).
    - ◇ Cannot use the key input to compress data.
- Blockcipher-based, like Davies-Meyer, used in SHA-2:
    - ◇ Exploit both key and plaintext inputs for compression.
    - ◇ Provably secure over $\mathbb{F}_2$, (PGV [14, 4]).



Sponge



Davies-Meyer

The PGV-LC mode

Inspired by the PGV modes, we introduce the PGV-LC mode:

- Underlying Blockcipher $E\colon \mathbb{F}_p^\kappa \times \mathbb{F}_p^n \to \mathbb{F}_p^n$.
- Matrix $\boldsymbol{R} \in \mathbb{F}_p^{\ell \times n}$ parametrizes output size.
  - ◇ Compresses its input $\Rightarrow \ell \leq n$.
  - ◇ Algebraic generalization of e.g. truncation and chopping.

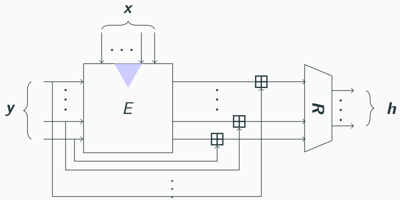


Figure 2: A depiction of the PGV-LC mode: $\boldsymbol{x} \in \mathbb{F}_p^\kappa$, $\boldsymbol{y} \in \mathbb{F}_p^n$, $\boldsymbol{h} \in \mathbb{F}_p^\ell$.

The PGV-ELC mode

We further generalize the design with the PGV-ELC mode:

- Matrices $\boldsymbol{K} \in \mathbb{F}_p^{\kappa \times \kappa'}$ and $\boldsymbol{P} \in \mathbb{F}_p^{n \times n'}$ parametrize input size.
  - ◇ Expand their inputs $\Rightarrow \kappa' \leq \kappa$ and $n' \leq n$.
  - ◇ Algebraic generalization of e.g. zero-padding.
- Matrix $\boldsymbol{F} \in \mathbb{F}_p^{\ell \times n'}$ adapts input to output size.
  - ◇ Expands its input $\Rightarrow \ell \leq n'$.

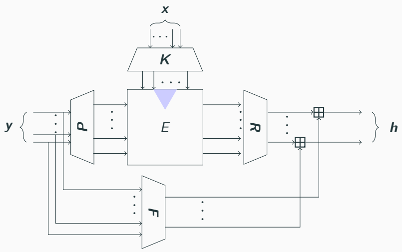


Figure 3: Visualization of PGV-ELC: $\boldsymbol{x} \in \mathbb{F}_p^{\kappa'}$, $\boldsymbol{y} \in \mathbb{F}_p^{n'}$, $\boldsymbol{h} \in \mathbb{F}_p^{\ell}$

How to Prove Your Security

In order to prove that our modes are secure, we need:

- A formal model: the ideal cipher $E \xleftarrow{\$} \mathsf{Block}\big(\mathbb{F}_p^\kappa, \mathbb{F}_p^n\big)$.
    - Standard security assumption in classic cryptography.
    - $\approx$ For permutations, ideal permutation $\pi \xleftarrow{\$} \mathsf{Perm}\big(\mathbb{F}_p^n\big)$.
- An adversary $\mathcal{A}$:
    - Unbounded randomized algorithm.
    - Can query E forward and backward via the oracle $\mathcal{E}$.
- A security notion (e.g. collision resistance).
- An advantage function $\mathbf{Adv}_{\mathrm{scheme}}^{\mathrm{NOTION}}(\mathcal{A}, q)$:
    - Must be negligible in the number q of oracle queries.
    - $\mathbf{Adv}_{\mathrm{scheme}}^{\mathrm{NOTION}}(q) = \max_{\mathcal{A}}\big\{\mathbf{Adv}_{\mathrm{scheme}}^{\mathrm{NOTION}}(\mathcal{A}, q)\big\}$

PGV-LC Security

Collision resistance:

$$\mathbf{Adv}_C^{\mathrm{COL}}(\mathcal{A}, q) = \Pr\Big[\big(\boldsymbol{x}, \boldsymbol{x}'\big) \xleftarrow{\$} \mathcal{A}^{\mathcal{E}}() : \boldsymbol{x} \neq \boldsymbol{x}' \wedge C_E(\boldsymbol{x}) = C_E\big(\boldsymbol{x}'\big)\Big]$$

For PGV-LC $C_E(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{R} \cdot (E_{\boldsymbol{y}}(\boldsymbol{x}) + \boldsymbol{x})$:

1. Consider $\boldsymbol{R}$ right-invertible (full row rank).
2. $\mathbb{F}_p^n$ is partitioned into p^ℓ equivalence classes.
3. $\mathcal{A}$ can exploit partition unbalances from oracle replies.
4. Still, $\mathbf{Adv}_C^{\mathrm{COL}}(q) \leq \frac{q^2+q}{p^\ell - q}$ (≈ birthday attack).
5. Similarly, for preimage resistance: $\mathbf{Adv}_C^{\mathrm{PRE}}(q) \leq \frac{q}{p^\ell - q}$.

PGV-ELC Security

Collision resistance:

$$\mathbf{Adv}_C^{\mathrm{COL}}(\mathcal{A}, q) = \Pr\Big[\big(\boldsymbol{x}, \boldsymbol{x}'\big) \xleftarrow{\$} \mathcal{A}^{\mathcal{E}}() : \boldsymbol{x} \neq \boldsymbol{x}' \wedge C_E(\boldsymbol{x}) = C_E\big(\boldsymbol{x}'\big)\Big]$$

For PGV-ELC $C_E(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{R} \cdot E_{\boldsymbol{Ky}}(\boldsymbol{Px}) + \boldsymbol{Fx}$:

1. Consider $\boldsymbol{K}$ and $\boldsymbol{P}$ left-invertible, $\boldsymbol{F}$ right-invertible.
2. Linear transformations induce partitions.
3. 'Meaningless' queries, cannot be used to form a collision:
    - However, can be exploited to guide further queries.
4. Nevertheless, we again obtain $\mathbf{Adv}_C^{\mathrm{COL}}(q) \leq \frac{q^2+q}{p^\ell - q}$.
5. Similarly, for preimage resistance: $\mathbf{Adv}_C^{\mathrm{PRE}}(q) \leq \frac{q}{p^\ell - q}$.

Merkle Tree Opening Security

Security notion for openings over a t-ary Merkle Tree:

- Merkle Tree intended as a hash function H.
- Generator $\mathcal{G}$ creates an opening π.
- Verifier $\mathcal{V}$ checks validity of π.
- Adversary $\mathcal{A}$ attempts to forge $\tilde{\pi}$.

Formally:

$$\mathbf{Adv}^{\mathrm{OPEN}}_{H,\mathcal{G},\mathcal{V}}(\mathcal{A}, q) =$$
$$\Pr\Big[M \xleftarrow{\$} \left(\mathbb{F}_p^m\right)^*, \tilde{\pi} \xleftarrow{\$} \mathcal{A}^{\mathcal{E}}(M) : \forall i \in \mathbb{N}\colon \tilde{\pi} \neq \mathcal{G}(M, i) \wedge \mathcal{V}(\tilde{\pi}, H_C(M)) = \top\Big]$$

Merkle Tree Opening Security (cont.)

For a t-ary Merkle Tree:

- $\mathbf{Adv}^{\mathrm{OPEN}}_{H,\mathcal{G},\mathcal{V}}(q) \leq \mathbf{Adv}^{\mathrm{COL}}_{C}(q)$
- Additionally, $\mathbf{Adv}^{\mathrm{COL}}_{H}(q) \leq \mathbf{Adv}^{\mathrm{COL}}_{C}(q) + \mathbf{Adv}^{\mathrm{PRE}}_{C}(q)$.
- Proof is standard, generalizes reasoning for binary trees.

$\Longrightarrow$ Our modes can be securely used for MT commitments.

Implementations and Experiments

Consider the POSEIDON hash function:

- Sponge mode over the fixed-key HADES block cipher.
- Affine key scheduler, which we instantiated with:

$$\boldsymbol{M}_{\mathcal{K},2} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \qquad \boldsymbol{M}_{\mathcal{K},4} = \begin{bmatrix} 2 & 3 & 1 & 1 \\ 1 & 2 & 3 & 1 \\ 3 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

- Instantied in PGV-LC: POSEIDON-DM.
- All other parameters kept the same as in POSEIDON.
  - ◇ Focus on improvement due to compression mode.

R1CS **arithmetization**

We considred R1CS arithmetization:

- Used by many ZK-SNARKs (Groth16, Aurora, Spartan, ...):
  - R1CS System: $\boldsymbol{Ax} \odot \boldsymbol{Bx} = \boldsymbol{Cx}$
- Concrete performance tends to follow theoretical numbers.

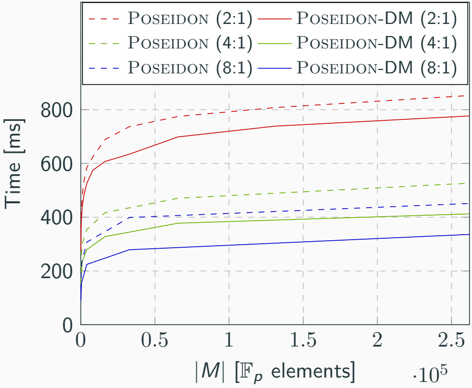
Hash	Compression Rate 2:1	4:1	8:1
POSEIDON	237	288	384
POSEIDON-DM	213	213	261
Constraint Reduction			
POSEIDON-DM w.r.t. POSEIDON	−11%	−35%	−47%

Table 1: Number of R1CS constraints for target primitives.

Benchmarks: Proof Generation

Time to generate a MT opening proof:

- Scalar field of the BLS12-381 elliptic curve: $\log_2(p) \approx 255$.
- ZK-SNARK framework: Groth16 (`libsnark`).



Benchmarks: Native execution

Native evaluation time speedup :

- Averaged over the scalar field of various curves:
  - BLS12-381, BN254, Ed-180.

Library	2:1	4:1	8:1
`NTL`	$1.17\times$	$2.80\times$	$2.51\times$
`libff`	$1.17\times$	$2.87\times$	$2.57\times$
`libarith`	$1.15\times$	$2.27\times$	$2.27\times$

Table 2: Poseidon-DM speed-up for a single compression call.

Benchmarks: Arity Matters

Choosing an optimal arity of the Merkle Tree matters:

- Binary trees are the most common choice.
- For generating an opening proof:
  - $8{:}1$ Poseidon-DM $\approx 2.5\times$ faster than $2{:}1$ Poseidon.
- For building the tree:
  - $4{:}1$ Poseidon-DM $\approx 4\times$ faster than $2{:}1$ Poseidon.
- 💡 Improve existing *t*-ary Merkle Tree opening proof circuits:
  - $\approx 10\%$ improvement to known strategies.

Roll credits

$\mathcal{T}he\ \mathcal{E}nd$

Thank you for your attention!

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